Design Step 5.5

STRESS IN PRESTRESSING STRANDS

Design Step 5.5.1

Stress in prestressing strands at nominal flexural resistance

The stress in prestressing steel at nominal flexural resistance may be determined using stress compatibility analysis. In lieu of such analysis a simplified method is presented in S5.7.3.1.1. This method is applicable to rectangular or flanged sections subjected to flexure about one axis where the Whitney stress block stress distribution specified in S5.7.2.2 is used and for which f_{pe}, the effective prestressing steel stress after losses, is not less than $0.5f_{pu}$. The average stress in prestressing steel, f_{ps}, may be taken as:

$$f_{ps} = f_{pu}[1 - k(c/d_p)] \qquad \text{(S5.7.3.1.1-1)}$$

where:

$$k = 2(1.04 - f_{py}/f_{pu}) \qquad \text{(S5.7.3.1.1-2)}$$

The value of "k" may be calculated using the above equation based on the type and properties of prestressing steel used or it may be obtained from Table SC5.7.3.1.1-1.

The distance from the neutral axis to the compression face of the member may be determined as follows:

for T-section behavior (Eq. S5.7.3.1.1-3):

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y - 0.85 \beta_1 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \dfrac{f_{pu}}{d_p}}$$

for rectangular section behavior (Eq. S5.7.3.1.1-4):

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}}$$

T-sections where the neutral axis lies in the flange, i.e., "c" is less than the slab thickness, are considered rectangular sections.

From Table SC5.7.3.1.1-1:

$k = 0.28$ for low relaxation strands

Assuming rectangular section behavior with no compression steel or mild tension reinforcement:

$$c = A_{ps}f_{pu}/[0.85f'_c\beta_1 b + kA_{ps}(f_{pu}/d_p)]$$

For the midspan section

Total section depth, h = girder depth + structural slab thickness
= 72 + 7.5
= 79.5 in.

d_p = h – (distance from bottom of beam to location of P/S steel force)
= 79.5 – 5.0
= 74.5 in.

β_1 = 0.85 for 4 ksi slab concrete (S5.7.2.2)

b = effective flange width (calculated in Section 2 of this example)
= 111 in.

c = 6.73(270)/[0.85(4)(0.85)(111) + 0.28(6.73)(270/74.5)]
= 5.55 in. < structural slab thickness = 7.5 in.
The assumption of the section behaving as a rectangular section is correct.

Notice that if "c" from the calculations above was greater than the structural slab thickness (the integral wearing surface is ignored), the calculations for "c" would have to be repeated assuming a T-section behavior following the steps below:

1) *Assume the neutral axis lies within the precast girder flange thickness and calculate "c". For this calculation, the girder flange width and area should be converted to their equivalent in slab concrete by multiplying the girder flange width by the modular ratio between the precast girder concrete and the slab concrete. The web width in the equation for "c" will be substituted for using the effective converted girder flange width. If the calculated value of "c" exceeds the sum of the deck thickness and the precast girder flange thickness, proceed to the next step. Otherwise, use the calculated value of "c".*

2) *Assume the neutral axis is below the flange of the precast girder and calculate "c". The term "$0.85 f'_c \beta_1 (b - b_w)$" in the calculations should be broken into two terms, one refers to the contribution of the deck to the composite section flange and the second refers to the contribution of the precast girder flange to the composite girder flange.*

f_{ps} = $f_{pu}[1 - k(c/d_p)]$ (S5.7.3.1.1-1)
= 270[1 – 0.28(5.55/74.5)]
= 264.4 ksi

Design Step 5.5.2 **Transfer and development length**

Transfer Length = 60(Strand diameter) (S5.11.4.1)
= 60(0.5 in.)
= 30 in.

Development Length = $\ell_d \geq \kappa[f_{ps} - (2/3)f_{pe}]d_b$ (S5.11.4.2-1)

From earlier calculations:
f_{ps} = 264.4 ksi (Design Step 5.4.8)
f_{pe} = 162.83 ksi (Design Step 5.5.1)

From S5.11.4.2, κ = 1.6 for fully bonded strands
From S5.11.4.3, κ = 2.0 for partially debonded strands

For fully bonded strands (32 strands):

$\ell_d \geq 1.6[264.4 - (2/3)162.83](0.5) = 124.7$ in. (10.39 ft. or 10'-4 11/16")

For partially debonded strands (two groups of 6-strands each):

$\ell_d \geq 2.0[264.4 - (2/3)162.83](0.5) = 155.8$ in. (12.98 ft. or 12'-11 ¾")

Design Step 5.5.3 **Variation in stress in prestressing steel along the length of the girders**

According to S5.11.4.1, the prestressing force, f_{pe}, may be assumed to vary linearly from 0.0 at the point where bonding commences to a maximum at the transfer length. Between the transfer length and the development length, the strand force may be assumed to increase in a parabolic manner, reaching the tensile strength of the strand at the development length.

To simplify the calculations, many jurisdictions assume that the stress increases linearly between the transfer and the development lengths. This assumption is used in this example.

As shown in Figures 2-5 and 2-6, each beam contains three groups of strands:

Group 1: 32 strands fully bonded, i.e., bonded length starts 9 in. outside the centerline of bearings of the noncomposite beam

Group 2: 6 strands. Bonded length starts 10 ft. from the centerline of bearings of the noncomposite beam, i.e., 10'-9" from the end of the beam

Group 3: 6 strands. Bonded length starts 22 ft. from the centerline of bearings of the noncomposite beam, i.e., 22'-9" from the end of the beam

For each group, the stress in the prestressing strands is assumed to increase linearly from 0.0 at the point where bonding commences to f_{pe}, over the transfer length, i.e., over 30 inches. The stress is also assumed to increase linearly from f_{pe} at the end of the transfer length to f_{ps} at the end of the development length. Table 5.5-1 shows the strand forces at the service limit state (maximum strand stress = f_{pe}) and at the strength limit state (maximum strand stress = f_{ps}) at different sections along the length of the beams. To facilitate the calculations, the forces are calculated for each of the three groups of strands separately and sections at the points where bonding commences, end of transfer length and end of development length for each group are included in the tabulated values. Figure 5.5-1 is a graphical representation of Table 5.5-1.

Table 5.5-1 – Prestressing Strand Forces

Dist. from Grdr End	Dist. from CL of Brg	Initial Prestressing Force at Transfer			
		Group 1	Group 2	Group 3	Total
(ft)	(ft)	(k)	(k)	(k)	(k)
0*	-0.75*	0.0			0.0
0.75	0.00	277.3			277.3
2.50	1.75	924.4			924.4
7.75	7.00	924.4			924.4
10.39	9.64	924.4			924.4
10.75**	10.00**	924.4	0.0		924.4
11.75	11.00	924.4	69.3		993.7
13.25	12.50	924.4	173.3		1,097.7
17.25	16.50	924.4	173.3		1,097.7
22.75***	22.00***	924.4	173.3	0.0	1,097.7
23.73	22.98	924.4	173.3	67.9	1,165.6
25.25	24.50	924.4	173.3	173.3	1,271.0
28.25	27.50	924.4	173.3	173.3	1,271.0
33.75	33.00	924.4	173.3	173.3	1,271.0
35.73	34.98	924.4	173.3	173.3	1,271.0
39.25	38.50	924.4	173.3	173.3	1,271.0
44.75	44.00	924.4	173.3	173.3	1,271.0
50.25	49.50	924.4	173.3	173.3	1,271.0
55.25	54.50	924.4	173.3	173.3	1,271.0
55.75	55.00	924.4	173.3	173.3	1,271.0
61.25	60.50	924.4	173.3	173.3	1,271.0
66.75	66.00	924.4	173.3	173.3	1,271.0
72.25	71.50	924.4	173.3	173.3	1,271.0
74.77	74.02	924.4	173.3	173.3	1,271.0
77.75	77.00	924.4	173.3	173.3	1,271.0
83.25	82.50	924.4	173.3	173.3	1,271.0
85.25	84.50	924.4	173.3	173.3	1,271.0
86.77	86.02	924.4	173.3	67.9	1,165.6
87.75+++	87.00+++	924.4	173.3	0.0	1,097.7
88.75	88.00	924.4	173.3		1,097.7
94.25	93.50	924.4	173.3		1,097.7
97.25	96.50	924.4	173.3		1,097.7
99.75++	99.00++	924.4	0.0		924.4
100.11	99.36	924.4			924.4
103.25	102.50	924.4			924.4
108.00	107.25	924.4			924.4
109.75	109.00	277.3			277.3
110.5+	109.75+	0.0			0.0

*, **, *** - Point where bonding commences for strand Groups 1, 2, and 3, respectively

+, ++, +++ - Point where bonding ends for strand Groups 1, 2, and 3, respectively

Table 5.5-1 (cont.) – Presstressing Strand Forces

Dist. from Grdr End	Dist. from CL of Brg	Prestressing Force After Losses				Force at the Nominal Flexural Resistance			
		Group 1	Group 2	Group 3	Total	Group 1	Group 2	Group 3	Total
(ft)	(ft)	(k)	(k)	(k)	(k)	(k)	(k)	(k)	(k)
0*	-0.75*	0.0			0.0	0.0			0.0
0.75	0.00	239.0			239.0	239.0			239.0
2.50	1.75	797.2			797.2	797.2			797.2
7.75	7.00	797.2			797.2	1,128.1			1,128.1
10.39	9.64	797.2			797.2	1,294.5			1,294.5
10.75**	10.00**	797.2	0.0		797.2	1,294.5	0.0		1,294.5
11.75	11.00	797.2	59.8		857.0	1,294.5	59.8		1,354.3
13.25	12.50	797.2	149.5		946.7	1,294.5	149.5		1,444.0
17.25	16.50	797.2	149.5		946.7	1,294.5	185.1		1,479.6
22.75***	22.00***	797.2	149.5	0.0	946.7	1,294.5	234.0	0.0	1,528.5
23.73	22.98	797.2	149.5	58.6	1,005.3	1,294.5	242.7	58.6	1,595.8
25.25	24.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	149.5	1,686.7
28.25	27.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	176.2	1,713.4
33.75	33.00	797.2	149.5	149.5	1,096.2	1,294.5	242.7	225.1	1,762.3
35.73	34.98	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
39.25	38.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
44.75	44.00	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
50.25	49.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
55.25	54.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
55.75	55.00	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
61.25	60.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
66.75	66.00	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
72.25	71.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
74.77	74.02	797.2	149.5	149.5	1,096.2	1,294.5	242.7	242.7	1,779.9
77.75	77.00	797.2	149.5	149.5	1,096.2	1,294.5	242.7	216.2	1,753.4
83.25	82.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	167.3	1,704.5
85.25	84.50	797.2	149.5	149.5	1,096.2	1,294.5	242.7	149.5	1,686.7
86.77	86.02	797.2	149.5	58.6	1,005.3	1,294.5	242.7	58.6	1,595.8
87.75+++	87.00+++	797.2	149.5	0.0	946.7	1,294.5	234.0	0.0	1,528.5
88.75	88.00	797.2	149.5		946.7	1,294.5	225.1		1,519.6
94.25	93.50	797.2	149.5		946.7	1,294.5	176.2		1,470.7
97.25	96.50	797.2	149.5		946.7	1,294.5	149.5		1,444.0
99.75++	99.00++	797.2	0.0		797.2	1,294.5	0.0		1,294.5
100.11	99.36	797.2			797.2	1,294.5			1,294.5
103.25	102.50	797.2			797.2	1,096.6			1,096.6
108.00	107.25	797.2			797.2	797.2			797.2
109.75	109.00	239.0			239.0	239.0			239.0
110.5+	109.75+	0.0			0.0	0.0			0.0

*, **, *** - Point where bonding commences for strand Groups 1, 2, and 3, respectively

+, ++, +++ - Point where bonding ends for strand Groups 1, 2, and 3, respectively

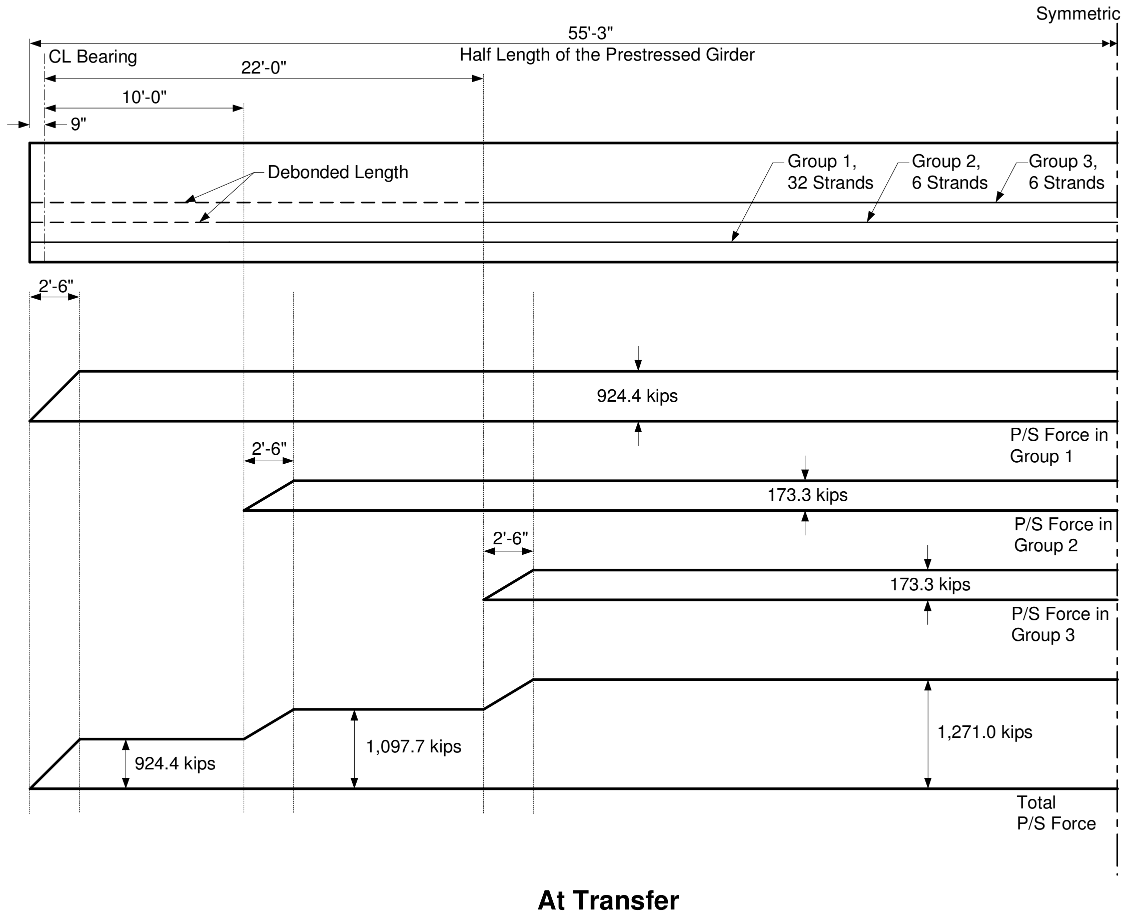
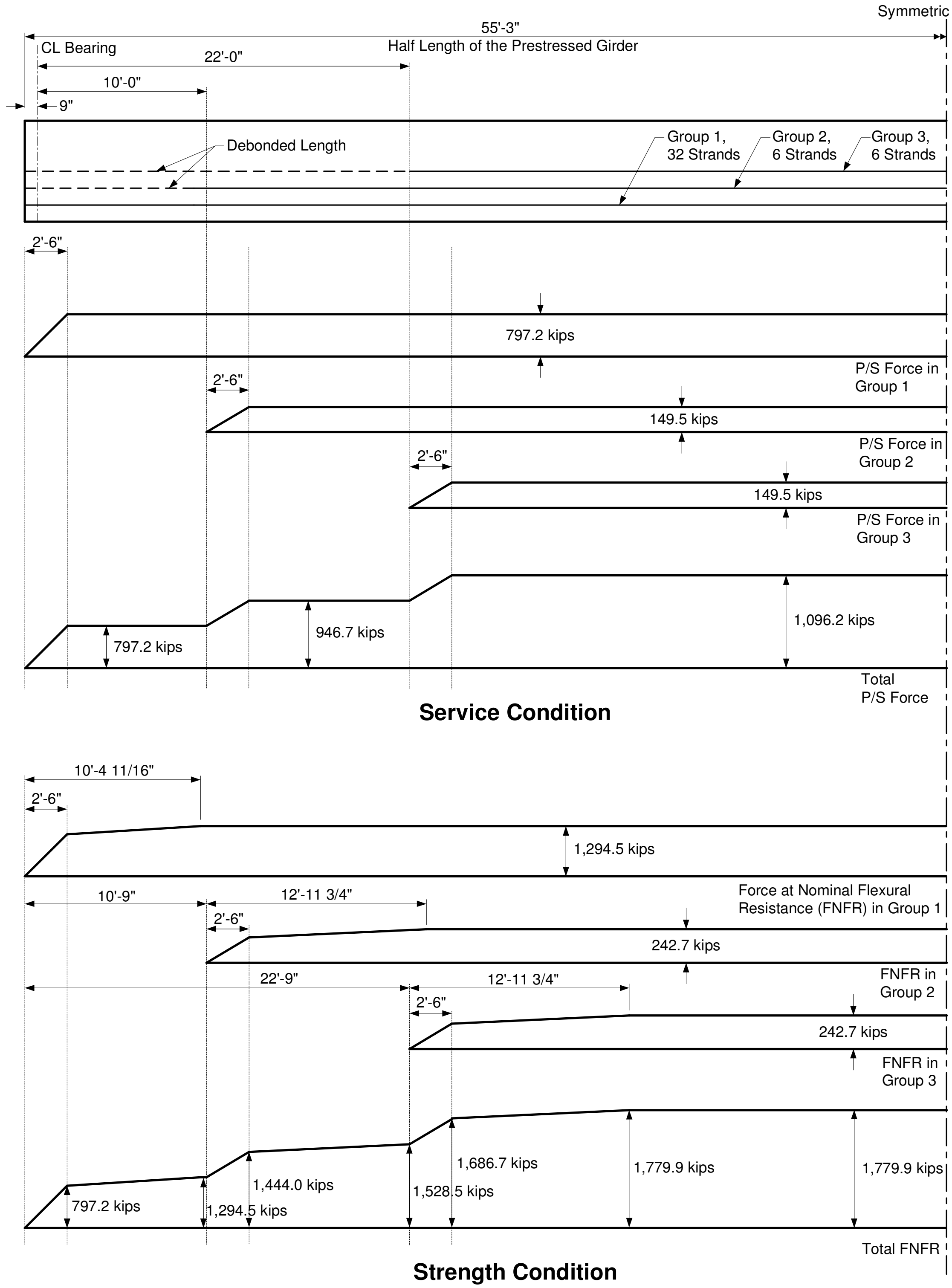


Figure 5.5-1 – Prestressing Strand Forces Shown Graphically



Transfer length = 30 in.
Development length, fully bonded = 124.7 in.
Development length, debonded = 155.8 in.

Figure 5.5-1 (cont.) – Prestressing Strand Forces Shown Graphically

Design Step 5.5.4

Sample strand stress calculations

Prestress force at centerline of end bearing after losses under Service or Strength

Only Group 1 strands are bonded at this section. Ignore Group 2 and 3 strands.

Distance from the point bonding commences for Group 1 strands = 0.75 ft < transfer length

Percent of prestressing force developed in Group 1 strands = 0.75/transfer length
= (0.75/2.5)(100) = 30%

Stress in strands = 0.3(162.83) = 48.8 ksi

Force in strands at the section = 32(0.153)(48.8) = 239 kips

Prestress force at a section 11 ft. from the centerline of end bearing after losses under Service conditions

Only strands in Group 1 and 2 are bonded at this section. Ignore Group 3 strands.

The bonded length of Group 1 strands before this section is greater than the transfer length. Therefore, the full prestressing force exists in Group 1 strands.

Force in Group 1 strands = 32(0.153)(162.83) = 797.2 kips

Distance from the point bonding commences for Group 2 strands = 1.0 ft. < transfer length

Percent of prestressing force developed in Group 2 strands = 1.0/transfer length
= (1.0/2.5)(100) = 40%

Stress in Group 2 strands = 0.4(162.83) = 65.1 ksi

Force in Group 2 strands at the section = 6(0.153)(65.1) = 59.8 kips

Total prestressing force at this section = force in Group 1 + force in Group 2
= 797.2 + 59.8 = 857 kips

Strands maximum resistance at nominal flexural capacity at a section 7.0 ft. from the centerline of end bearing

Only Group 1 strands are bonded at this section. Ignore Group 2 and 3 strands.

Distance from the point bonding commences for Group 1 strands, i.e., distance from end of beam = 7.75 ft. (7'- 9")

This distance is greater than the transfer length (2.5 ft.) but less than the development length of the fully bonded strands (10.39 ft.). Therefore, the stress in the strand is assumed to reach f_{pe}, 162.83 ksi, at the transfer length then increases linearly from f_{pe} to f_{ps}, 264.4 ksi, between the transfer length and the development length.

Stress in Group 1 strands = 162.83 + (264.4 – 162.83)[(7.75 – 2.5)/(10.39 – 2.5)]
= 230.41 ksi

Force in Group 1 strands = 32(0.153)(230.41)
= 1,128.1 kips

<u>Strands maximum resistance at nominal flexural capacity at a section 22 ft. from centerline of end bearing</u>

Only strands in Group 1 and 2 are bonded at this section. Ignore Group 3 strands.

The bonded length of Group 1 strands before this section is greater than the development length for Group 1 (fully bonded) strands. Therefore, the full force exists in Group 1 strands.

Force in Group 1 strands = 32(0.153)(264.4) = 1,294.5 kips

The bonded length of Group 2 at this section = 22 – 10 = 12 ft.

Stress in Group 2 strands = 162.83 + (264.4 – 162.83)[(12 – 2.5)/(12.98 – 2.5)]
= 254.9 ksi

Force in Group 2 strands = 6(0.153)(254.9) = 234.0 kips

Total prestressing force at this section = force in Group 1 + force in Group 2
= 1,294.5 + 234.0
= 1,528.5 kips